ECS 452: Exercise 4 Solution

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former groups.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Name	ID
Prapun	555

1. For each of the following DMC's probability transition matrices **Q**, (i) indicate whether the corresponding DMC is weakly symmetric (Yes or No), (ii) evaluate the corresponding capacity value (your answer should be of the form X.XXXX), and (iii) specify the channel input pmf (a row vector **p**) that achieves the capacity.

Check that
(1) all the rows of Q are permutations of each other
(2) all the column sums are equal

$H(\underline{r}) = H([\frac{1}{4}\ \frac{1}{2}\ \frac{1}{4}])$
$= -2\times\frac{1}{4}\log_2\frac{1}{4} - \frac{1}{2}\log_2\frac{1}{2}$
$= 1+\frac{1}{2} = 1.5$

$H(\underline{r}) = H([\frac{2}{3}\ \frac{1}{3}])$

Q	Weakly Symmetric?	C	p
$\begin{bmatrix}1/4 & 1/2 & 1/4\\ 1/2 & 1/4 & 1/4\\ 1/4 & 1/4 & 1/2\end{bmatrix}$	(1) ✓ (2) ✓ Yes	$\log_2\lvert\mathcal{Y}\rvert - H(\underline{r})$ $=\log_2\lvert 3\rvert - 1.5$ ≈ 0.0850 [bpcu]	C is achieved by uniform X on $\mathcal{X}$ $\underline{p}^* = [\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}]$
$\begin{bmatrix}0 & 1/2 & 1/2\\ 1 & 0 & 0\end{bmatrix}$	(1) ✗ (2) ✗ No	Note that there is only one non-zero element in each column ⇒ This is NO^2 channel ⇒ $C = \log_2\lvert\mathcal{X}\rvert$ $=\log_2\lvert 2\rvert$ $= 1$ [bpcu]	⇒ C is achieved by uniform X $\underline{p}^* = [\frac{1}{2}\ \frac{1}{2}]$
$\begin{bmatrix}2/3 & 1/3 & 0\\ 0 & 1/3 & 2/3\end{bmatrix}$	(1) ✓ (2) ✓ Yes	$\log_2\lvert\mathcal{Y}\rvert - H(\underline{r})$ $=\log_2 3 + \frac{1}{3}\log_2\frac{1}{3} + \frac{2}{3}\log_2\frac{2}{3}$ $=\cancel{\log_2 3} - \frac{1}{3}\cancel{\log_2 3} + \frac{2}{3}\log_2 2 - \cancel{\frac{2}{3}\log_2 3}$ $= 2/3$ [bpcu]	C is achieved by uniform X on $\mathcal{X}$ $\underline{p}^* = [\frac{1}{2}\ \frac{1}{2}]$
$\begin{bmatrix}1/4 & 1/2 & 1/4\\ 1/4 & 1/2 & 1/4\end{bmatrix}$	(1) ✓ (2) ✗ No	Note that all the rows of Q are the same ⇒ $Q(y\mid x)$ does not depend on x ⇒ $X \perp\!\!\!\perp Y$ ⇒ $I(X;Y) = 0$ for any $p(x)$ ⇒ $C = 0$ [bpcu]	Any $\underline{p}^*$ will give the same $I(X;Y) = C = 0$.

ECS 452: Exercise 5 **Solution**

Instructions

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5. **Do not panic.**

Name	ID
Prapun	**555**

1. Consider a block code whose generator matrix is

$$\mathbf{G} = \left[\overbrace{\begin{matrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{matrix}}^{I_3}\;\overbrace{\begin{matrix}1 & 0 & 1\\0 & 1 & 1\\1 & 1 & 0\end{matrix}}^{P}\right]$$

a. Is it a linear code?

Yes, any code that has a generator matrix is linear.

b. Find the code length n

The #columns of G is 6

c. Find the code dimension k

The #rows of G is 3

d. Find the parity check matrix **H** of this code.

$$H = \left[\overbrace{\begin{matrix}1 & 0 & 1\\0 & 1 & 1\\1 & 1 & 0\end{matrix}}^{P^T}\;\overbrace{\begin{matrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{matrix}}^{I}\right]$$

e. Suppose we receive $\underline{\mathbf{y}}$ = 101101.

i. Find the syndrome vector $\underline{\mathbf{s}}$

$$\underline{s} = \underline{y}\,H^T = \left[\begin{pmatrix}1\\0\\1\end{pmatrix}+\begin{pmatrix}1\\1\\0\end{pmatrix}+\begin{pmatrix}1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\1\end{pmatrix}\right]^T = \begin{pmatrix}1\\1\\0\end{pmatrix}^T = [1\,1\,0]$$

ii. Find $\hat{\underline{\mathbf{x}}}$ and $\hat{\underline{\mathbf{b}}}$.

The syndrome $\underline{s}$ is the same as the third column of H.

Therefore, $\hat{\underline{e}} = [0\;0\;1\;0\;0\;0]$ and

$$\hat{\underline{x}} = \hat{\underline{y}} - \hat{\underline{e}} = \hat{\underline{y}} \oplus \hat{\underline{e}} = [1\,0\,0\,1\,0\,1]$$

$$\hat{\underline{b}} = [1\,0\,0]$$

From G, we have I_3 in the front, so the message $\underline{b}$ will be the first three bits of the codeword $\underline{x}$.

ECS 452: Exercise 6 **Solution**

Instructions

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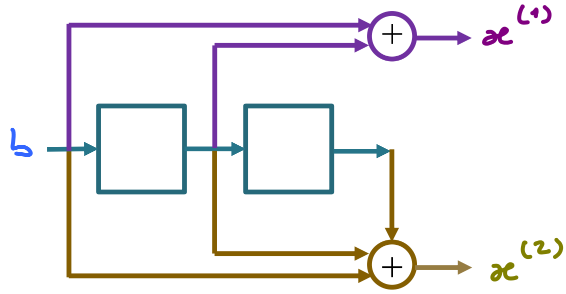
Name	ID
Prapun	555

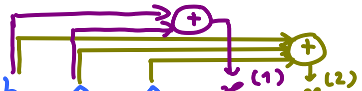
1. Consider a convolution encoder represented by the following diagram



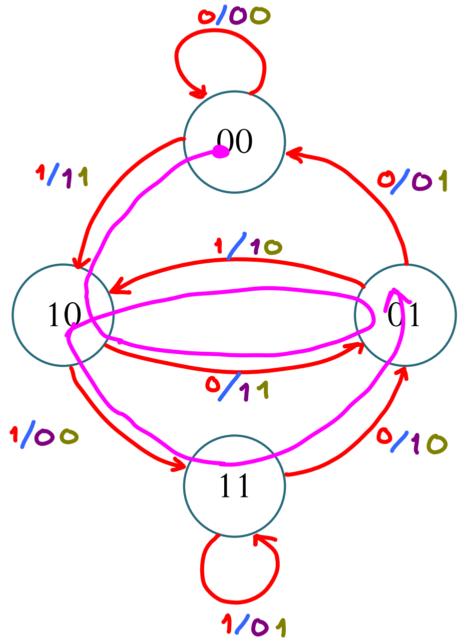
a. Draw the corresponding state (transition) diagram

First, observe that this encoder use a shift register with two FFs which is the same as in the lecture. Therefore, the arrows will be the same as what we had in the lecture

Note, however, that the connections that produce the outputs are different from the encoder in lecture.



b	s_0	s_1	$x^{(1)}$	$x^{(2)}$
0	0	0	0	0
1	0	0	1	1
0	0	1	0	1
1	0	1	1	0
0	1	0	1	1
1	1	0	0	0
0	1	1	1	0
1	1	1	0	1



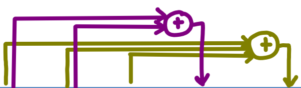
b. Suppose the information input bits (the message bits) are 10110. Find the corresponding codeword **x**

i. by using the direct method (filling out the table below)

and

ii. by "tracing" the corresponding path on the state diagram above

see the trace in the diagram above



b	s_0	s_1	$x^{(1)}$	$x^{(2)}$
1	0	0	1	1
0	1	0	1	1
1	0	1	1	0
1	1	0	0	0
0	1	1	1	0

$\underline{x}$ = [1 1 1 1 1 0 0 0 1 0]